MATHEMATICAL MODELLING OF THE REACTION LATENCY.
PART III: A MODEL OF AVOIDANCE REACTION LATENCY
AND AVOIDANCE LEARNING¹

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Abstract. The linear-dynamic-stochastic model of a reaction latency as applied to avoidance experiment is presented. Reactions are classified on the basis of the model into following classes: escape, avoidance, late avoidance, three types of inter-trial responses and “no reaction”. The experimental latency distribution is split into latency distributions of the escape, avoidance and late avoidance responses, providing a new insight into latency distribution. The results of fitting the model to latency measurements obtained in the avoidance conditioning experiment are presented. The same processes of the parameter changes as in the escape conditioning are discovered, one causing a latency to decrease and the other causing a latency to increase during learning. The first process affects a latency stronger than the second and, consequently, the latency decreases during learning. The second process is responsible for a decay of inter-trial-responses during experiment. The value of the correlation coefficient between the threshold of avoidance reaction and the threshold of escape reaction was also estimated. Negative values of this coefficient were obtained, therefore, on the average, the greater the avoidance reaction threshold the smaller the escape one. In the course of learning the correlation coefficient tends to be equal to $-1$, i.e., as a result of training, both thresholds became dependent in a functional (non-random) way. This result may provide an objective index of the “state of learning”. The model provides a new tool for analysis of results of latency-based experiments.

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INTRODUCTION

The goal of this paper is to describe the application of the dynamic threshold model of reaction latency to the latency of the avoidance reaction. The basic idea of the dynamic stochastic model (which has been described in an earlier paper (2)) is that a stimulus causes changes in an excitatory potential which acts on a threshold-type decision system. The linear dynamic stochastic (LDS) model has been previously applied to the escape reaction and gave results consistent with a set of experimental data (see (3)). The present avoidance experiment differs from the earlier escape experiment in that two stimuli are applied. In consequence, substantial parts of the dynamic model (particularly the transformation system and the decision system) are more complicated for the avoidance experiment than for the escape experiment. In this paper, the biological aspects of the special "random threshold" (RT) version of the linear, dynamic, stochastic model of the avoidance reaction are discussed. This version has been identified\(^2\) using real experimental data, and the results of the identification are presented and discussed in this paper.

THE MODEL STRUCTURE

The general LDS model structure and the model equations have been discussed in an earlier work (2) and only the results are presented here. For the avoidance experiment we have two stimuli: conditioned and unconditioned. The second one is applied if and only if the animal does not answer properly to the first (conditioned) stimulus in a predetermined period of time. It is assumed that each stimulus acts on an adequate transformation system. Each transformation system has the same equation

\[
dE_i(t) = [k_iS_i(t) - a_iE_i(t)]dt + dw_i(t),
\]

\[
E_i(t_0) = E_{0i}, \quad i \in \{u, c\},
\]

where \(i\) denotes an index of the system: \(u\) (for unconditioned) or \(c\) (for conditioned)\(^3\), \(E_i(t)\) denotes the excitatory function, \(w_i\) denotes the Brownian motion, \(S_i\) denotes the external stimulus, \(k\) and \(a\) are parameters of the model: the static gain and the dynamic gain, respectively. The time measured from beginning of the conditioned stimulus is denoted by \(t\). The initial conditions are set at time 0 for the equation for the conditioned stimulus and at time \(T\) (\(T\) is equal to delay in applying the unconditioned stimulus) for the equation for the unconditional stimulus (i.e., \(t_{pc} = 0, t_{uc} = T\)).

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\(^2\) By the "identification" of the model we mean the estimation of all unknown quantities used in the model.

\(^3\) The following indexation is used: \(c\) for quantities connected with the conditioned system, \(u\) for quantities connected with the unconditioned system and the index \(i\) denotes one of these.
A stochastic differential equation (1) is not usually used by biologists but in a special case to be discussed below it can be simplified to a more common form of differential equation. There are two sources of randomness in equation (1): Brownian motion, and initial conditions \( E_{oc} \) and \( E_{ou} \).

![Diagram](image)

**Fig. 1.** Reaction latency as the first moment in which one of excitatory potentials exceeds for the first time the corresponding threshold level.

After the stimulus is received the outputs from both transformation systems are compared and the decision about the external reaction is made in the next part of the system, the decision system. A very simple decision system is assumed in the LDS model: the external reaction occurs when any of the excitatory signals reaches its threshold value (which is Gaussian random variable) for the first time for a given stimulation (Fig. 1). Suppose that the variable \( y(t) \) is related to the performance of the external reaction: \( y(t) = 0 \) denotes lack of the reaction, \( y(t) = Y \) denotes performance of the reaction (by the "reaction" we mean the reaction desired by the experimenter). Consequently

\[
y(t) = \begin{cases} 
0 & \text{if } E_c(t) < L_c \text{ and } E_u(t) < L_u, \\
Y & \text{if } E_c(t) \geq L_c \text{ or } E_u(t) \geq L_u, 
\end{cases}
\]

where \( L = \begin{bmatrix} L_c \\ L_u \end{bmatrix} \) is a Gaussian random vector, with the expectation vector \( \mathbb{E}L \) and the covariance matrix \( \mathbb{V}L \), namely,
The identification of the full model is extremely complicated and cannot be performed from the small data bases available. In the sequel we assume that thresholds are the main source of randomness in the model because this assumption proved successful for modelling the escape reaction \((3)\). This version of the model will be called the RT-case (random threshold-case). It has also been assumed (see \((2)\) for a discussion) that \(\phi E_i(t_{io}) = 0\) (for \(i = u\) or \(c\)); as a consequence, we can put \(E_{oc} = 0\), \(E_{ou} = 0\). Under these assumptions we rewrite equation \((1)\) in the following form:

\[
\dot{E}_i(t) = k_i s_i(t) - a_i E_i(t),
E_i(t_{io}) = 0, \quad i \in \{u, c\}.
\]  

Here, each excitatory potential grows exponentially after the corresponding stimulus has begun, and each excitatory potential is equal to zero before this moment, i.e., (for \(a > 0\)):

\[
E_c(t) = \begin{cases} 
0 & \text{for } t < 0, \\
k_c s_c a_c^{-1} \exp(-a_c t) & \text{for } t \geq 0,
\end{cases}
\]

\[
E_u(t) = \begin{cases} 
0 & \text{for } t < T, \\
k_u s_u a_u^{-1} \exp(-a_u(t-T)) & \text{for } t \geq T.
\end{cases}
\]

The shape of the excitatory potentials in Fig. 1 illustrates this result.

By eliminating the time \(t\) from Eqs. \((5)\) and \((6)\) we can present \(E_u\) as a function of \(E_c\), namely

\[
E_u = \begin{cases} 
0 & \text{for } 0 \leq E_c \leq b_0, \\
k_u s_u a_u^{-1} - b(k_c s_c a_c^{-1} - E_c)^a_{uc} & \text{for } b_0 < E_c < k_c s_c a_c^{-1},
\end{cases}
\]

where

\[
b_0 = k_c s_c a_c^{-1}(1-\exp(-a_c T)), \\
b_1 = k_u s_u a_u^{-1}(k_c s_c a_c^{-1})^a_{uc} \exp(a_c T).
\]

Let the vector function \(E(t)\):

\[
E(t) = \begin{bmatrix} E_c(t) \\ E_u(t) \end{bmatrix}
\]

correspond to the state of excitation caused by the stimuli.
The graph of this function will be called the excitatory curve. Each point on this curve corresponds to some time, i.e., for each time \( t \) we can find a point \((E_c(t), E_u(t))\) which lies on this curve.

![Excitatory Curve Diagram](image)

**Fig. 2.** The excitatory curve on a decision plane. The convexity of the power part of the curve depends on \( a_u \) and \( a_c \) (drawn for \( a_u > a_c \)).

Therefore (Fig. 2) the section of the excitatory curve which corresponds to parameters \( t \) less than \( T \) is a segment parallel to the \( E_c \) axis. The section of this curve which corresponds to parameters greater than \( T \) is the segment of the power function (the convexity of which depends on the relation between \( a_c \) and \( a_u \)). Therefore the straight section of the excitatory curve is related to the conditioned stimulation and the next section to the stimulation by both stimuli.

From the Eqs. (5) and (6) using the assumptions (2) and (3) one can find the distribution of the reaction latency \( \tau \). Note that

\[
\tau = \min(\tau_u, \tau_c), \tag{9}
\]

where \( \tau_u \) and \( \tau_c \) are the (correlated) latencies of the unconditioned and the conditioned reaction, respectively. Their distributions can be found by transforming the random variable \(
\begin{bmatrix}
L_u \\
L_c
\end{bmatrix}
\) to the random variable \(
\begin{bmatrix}
\tau_u \\
\tau_c
\end{bmatrix}
\), just as was done for the escape RT model. Consequently one can find the distribution function \( F_\tau \) of the reaction latency:

\[
F_\tau(t) = G(\tau_u(t), \tau_c(t)) \tag{10}
\]
where \( \varphi \) denotes the distribution function of the bivariate normal distribution with the expectation vector \( m_L \) and the covariance matrix \( \Sigma_L \) (Eq. (3)).

As follows from Eqs. (10) and (11), the distribution of the latency at time \( t \) (that is, the probability that the latency is less than \( t \)) can be found by integrating the bivariate normal distribution over the region as given in Fig. 3. This integration can only be done numerically and presents one of the numerical problems of the identification. It must be stressed that the latency distribution is not continuous (because its distribution function has a discontinuity point at the origin). Moreover, it does not tend to 1 when the argument tends to infinity (so-called improper distribution). These facts will be explained later.

\[
G(x, y) = \varphi(x, \infty) + \varphi(\infty, y) - \varphi(x, y),
\]

(11)

Let us consider the region \( R \) on the plane (Fig. 4):

\[
R = \{ (e_c, e_u) : e_c > L_c \text{ or } e_u > L_u \}. 
\]

(12)

Note that if at a certain time \( t \) the excitatory curve lies in region \( R \) then at least one of the excitatory potentials exceeded the corresponding threshold, in other words, the reaction must have occurred before the time \( t \). Consequently, the time at which the excitatory curve enters the region \( R \) is the time when the reaction occurs. Points
outside the region $R$ correspond to the times before the reaction has occurred. Therefore, the region $R$ will be called the reaction region, and the plane in which it lies, the decision plane. Note that because the threshold is a random variable and corresponds to the position of the corner of the reaction region $R$, this region has a "random position" on the plane.

The block diagram of the RT model is given in Fig. 5.

![Fig. 4. Decision plane and the reaction region.](image)

**Fig. 5.** The linear, dynamic, stochastic model in random-threshold version, for the avoidance reaction. The peak of the reaction region $R$ has been translated to the origin through subtraction of the values of the thresholds to show the role of the thresholds explicitly.
BIOLOGICAL INTERPRETATION

In this part we consider biological implications of the model starting with a classification of the reaction types predicted by the model. We assumed that the external reaction occurs when at least one of the excitatory potentials reaches a given threshold (which means that the excitatory curve crosses the border of the reaction region on the decision plane). Let us suppose that all the model parameters are given and let us consider the possible thresholds (i.e., the possible positions of the reaction region).

Note that the relationship between the reaction region and the excitatory curve determines which threshold has been crossed (i.e., which of the stimuli actually caused the reaction). If the excitatory curve crosses the "vertical" "part of the reaction region border, then the "conditioned" excitatory potential $E_c$ crosses the corresponding threshold before the "unconditioned" excitatory potential $E_u$ crosses its threshold; consequently, the "conditioned type" of the reaction has been performed. In the reverse situation, i.e., if the "horizontal" part of the reaction region border is crossed, the "unconditioned type" of the reaction is performed (Fig. 6). In the above, the term "crosses" can also denote a situation when the excitatory curve exceeds one (or both) threshold in the moment of the beginning of the stimulation, (i.e., $t = 0$). Such a situation will be referred to an inter-trial response. The possible positions of the reaction region are illustrated in Fig. 7. One can observe 7 types of the reaction. Let us analyze the types of reactions more carefully.

There are two types of the "unconditioned-type" reactions. The first (denoted by E) is a common escape reaction, and the second (denoted by ITR(E)) is, in fact, an inter-trial-response reaction because the unconditioned threshold is being exceeded in the moment zero, that is, without the stimulation.

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Fig. 6. Basic types of reactions predicted by the LDS-RT model. a, conditioned reactions, when the excitatory curve enters the reaction region through the vertical border (conditioned threshold). b, unconditioned reactions, when the excitatory curve enters the reaction region through the horizontal border (unconditioned threshold).
Next, we have three types of the "conditioned-type" reactions. There is common avoidance reaction (denoted by A) when the "conditioned" threshold is crossed during the stimulation by the conditioned stimulus. The second type consists in crossing the "conditioned" threshold during the period of stimulation by both stimuli. In other words, during the stimulation by conditioned and unconditioned stimuli, only conditioned excitatory potential reaches the corresponding threshold. This situation has been called "late avoidance" (LA) response and is in agreement with biological considerations. Actually, such a response cannot be distinguished from "common" escape in the analysed experiment, but, by means of the model, its probability can be calculated. It is possible to plan an experiment which verifies such calculations. The third type of the conditioned-type response is analogous to ITR(E), i.e., the conditioned threshold is exceeded before the moment of the conditioned stimulus onset. This type, which can also be related to the inter-trial-response will be denoted by ITR(A).

![Fig. 7. Classification of types of the reaction predicted by the LDS-RT model.](image)

The next type of response is also of ITR-type, and corresponds to a situation in which, in the moment of the beginning of stimulation, both threshold are exceeded. This will be called ITR(A×E) reaction.
The above three types of ITR reactions are indistinguishable in a common type of the experiment. They also cannot be distinguished from other types of ITR-reaction, that is, from a "proper" answer on the "improper" stimuli. In fact, in terms of the model, an ITR reaction requires a stimulus only to initiate thresholds setting. Naturally, this can also be made by another similar (or even unsimilar) stimulus. The problem of ITR-type responses is beyond the scope of this paper, and can be more simply presented in terms of the full LDS model as an effect related to Brownian fluctuation, avoiding a slightly sophisticated explanation given above.

The very last type of the reaction is, actually, a lack of the reaction: the situation which occurs if asymptotic values of the excitatory curves are too low to cross the adequate thresholds. For homogeneity, this situation will be counted as a particular reaction and called the NO-type of reaction.

The above considerations lead to a classification of the values of thresholds. Note that the position of the reaction region is designated by the threshold values, so it is identified with a type of the reaction (for a given excitatory curve).

In Fig. 8 the decision plane has been partitioned into 7 regions; the position of the threshold in the particular region of the plane designates the type of the reaction. According to the earlier remarks, a probability of each type of the reaction is equal to the integral over the corresponding region of the bivariate normal density, with parameters given by Eq. (3). One can also compute the probability of the reaction in cer-

![Fig. 8. Partition of the decision plane into regions corresponding to various types of reaction (according to the classification as given in Fig. 7). The shaded region corresponds to the reaction which latency is in a given interval \((t_1, t_2)\). This region contains two parts corresponding to possible types of this reaction (LA part and E part).](image-url)
tain interval \((t_1, t_2)\), as well as the probability that in this interval the reaction will be of a certain type. For instance, the region of integration required to compute probability \(P(t_1 \leq \tau < t_2)\) is shown in Fig. 8. This region is composed of two parts; integrating over each part gives the probability of the corresponding (LA or E) type of the reaction. The method can be easily understood when compared with Fig. 3 and using a relation \(P(t_1 \leq \tau < t_2) = F_\tau(t_2) - F_\tau(t_1)\).

**REMARKS ON IDENTIFICATION AND VERIFICATION**

Problems with identification of the LDS-RT model for the avoidance experiment are much more complicated than those for the escape experiment. The computations are more involved and the problem of small data sets is more crucial.

Let us characterize the identification problem briefly. There is one group of parameters connected with conditioned stimulus: \(k_c, s_c, a_c, m_{Le}, \sigma_{Le}\), another group connected with unconditioned stimulus: \(k_u, s_u, a_u, m_{Lu}, \sigma_{Lu}\), and a coupling parameter \(\varrho_L\). Several parameters must be assumed because it is impossible to identify them; namely, the scale units of the excitatory potentials, and the scale units of the stimuli. It is most convenient to choose the scales of the excitatory potentials equal to corresponding dispersions \(\sigma_{Lu}\) and \(\sigma_{Le}\). The scale units of the external stimulus can be set equal to 1. Consequently one can compare only relative values of the static gains obtained for different stimuli because they are expressed in different units (equal to a unit of the excitatory potential/unit of the conditioned stimulus).

Consequently, seven parameters: \(k_L, a_c, m_{Le}, k_u, a_u, m_{Lu}, \varrho_L\) must be estimated. The method of estimation is based on a minimization of \(\chi^2\) statistics connected to the theoretical and the experimental latency distributions. Some problems are connected with computing the theoretical distributions. These computations should have been fast because they had to be performed hundreds of times in an iterative minimization procedure. These problems are beyond the scope of this paper and will be presented in another work (see also (1)).

After estimation of the model parameters one can compute all a posteriori probabilities of various types of the reactions, the performance levels, ITR probabilities and so on. The shape of excitatory potentials, excitatory curve, reaction region can also be easily identified. For the adequate relations, the reader is referred to another work (1).

Testing of the avoidance model quality is more complicated than that for the escape model. As it was mentioned in the earlier work (3), the \(\chi^2\)-statistics do not have in this case the \(\chi^2\)-distribution. Still a comparison of the corresponding distributions can be made, and also the biological validation of the results remain. The model has been applied to analysis of the data obtained during an avoidance learning experiment. The results are described in the next section.
APPLICATION OF THE MODEL TO THE ANALYSIS OF AN AVOIDANCE LEARNING PROCESS

The training of the avoidance response, from which the data has been used in the model identification, was performed on cats in the Nencki Institute of Experimental Biology by Zielinski (4, 5). Cats were trained to avoid an electric shock; the conditioned stimulus was an acoustic white noise with the intensity of 50 dB (SPL) or (in the second group of animals), 70 dB (SPL). The electric current was adjusted according to the

Fig. 9. Sample latency distribution for consecutive stages of learning. The unconditioned stimulus has been applied in 25th time unit (1 time unit = 0.2 s), a) Group 70 dB, b) Group 50 dB. Numbers 1–5 refer to stages of learning.
animal sensitivity. The experiment consisted of a series of trials; in each trial, the conditioned stimulus was applied first during 25 time units (each unit equal to 0.2 s). If the animal did not answer during this time by pressing the bar, the electric current source was activated and both stimuli acted together until the time of a bar-pressing response. The experiment with particular animal lasted until an assumed level of performance was reached, measured by the number of avoidance reactions during a given number of trials. The reaction latency was measured in consecutive trials, and the latency series is a numerical result of the experiment. The length of this series depends on the animal.

To analyse these data by means of our model, the learning process in a given animal has been divided into stages of equal length (the Vincent division). According to the earlier experience with the escape data, the division into 5 stages has been accomplished. Because the length of the latency data series varies from animal to animal, the number of data in each stage is different for different animals. The data which belongs to a certain stage of the learning from all animals in one group are collated to enlarge sample size. Of course, this method leads to some degradation of estimates quality because of a great between-subject variability of latencies. In Fig. 9 the experimental latency distribution is given for all five stages of the learning process for each group of animals. One can easily see an effect of onsetting the unconditioned stimulus. These distributions were used in the identification.

An additional problem arose which concerns the identification of the "conditioned" parameters for the first stage of the learning process and the "unconditioned" parameters for the last stage. There was a relatively small amount of data for these stages and consequently, quality of the corresponding estimates is relatively poor. In the sequel we omit these results (1st stage for conditioned stimulus, 5th or even 4th stage for the unconditioned one).

Fig. 10. The changes in the static gains $k_c$ (Fig. 10a) and $k_u$ (Fig. 10b) during learning. Filled circles — results for the 70 dB group. Empty circles — results for the 50 dB group. A dashed line links points identified on the basis of too small amounts of data which therefore, are unreliable.
We now begin a discussion of the results with remarks on the parameters of the excitatory systems. Generally, they change during the learning just as in the escape training. The static gains (Fig. 10) increase for both stimuli (conditioned and unconditioned ones) which results in a faster growth of the corresponding excitatory potentials. The dynamic gains also increase during learning (Fig. 11). In the result, the asymptotic level of the conditioned excitatory potential $E_c(\infty)$ increases during learning (Fig. 12a); the analogous quantity $E_u(\infty)$ for unconditioned excitatory potential is smaller but also increases (Fig. 12b).

The changes of the transformation systems must be compared with the corresponding changes of the decision systems. The mean values of the thresholds (Fig. 13)
became higher during the experiment for both stimuli. (The only exception is 50 dB “conditioned” threshold which remains approximately constant). As a consequence of this growth, the probability of the inter-trial responses decreases during the learning (Fig. 14). Also, in the effect of the parallel growth of the asymptote of the excitatory potential, the difference $H = E_i(\infty) - m_{LI}$, $i = c$ or $u$ (Fig. 15), remain approximately constant for the conditioned stimulus and even decreases for the unconditioned one. This difference is related to the ultimate level of performance of a certain (conditioned or unconditioned) type of the reaction, i.e., the probability of the adequate (conditioned or unconditioned) type of the reaction during the infinitely long (in practice: about 100 units, i.e., 20 s) stimulation. The quantity $H_c$ (for the conditioned stimulus) remains constant beginning at the second stage of learning. In the result, the latency of the conditioned response decreases during this period but a level of “conditioned” performance $H_c$ does not change.

![Fig. 13. The changes in the mean threshold value $m_{Le}$ (Fig. 13a) and $m_{Lu}$ (Fig. 13b) during learning. For explanations see Fig. 10.](image)

The analogous quantity, $H_u$ (for the unconditioned stimulus) decreases during learning. In the result, the latency of unconditioned response decreases during learning but a level of “unconditioned” performance $H_u$ also decreases what can be interpreted as an effect of the extinction of the unconditioned responding.

Note that the notion “level of performance” is also in use and denotes in this case the percentage of proper responses during conditioned stimulation. This quantity is readily smaller than our “level of conditioned performance” because it does not count in the late-avoidance responses. The level of performance can easily be read-out and compared with the analogous quantity estimated on the basis of the model. The comparison (Fig. 16) shows good correlation between the model and the experimental data.

One can also reconstruct the theoretical latency distribution function on the basis of the estimated parameters. This distribution, however, contains inter-trial response
probability as a probability of "zero" latency. This probability cannot be compared with experimental results because ITR-level was not measured rigorously. Comparison of the theoretical and experimental distribution functions shows their consistency after shifting the theoretical distribution function to the origin (Fig. 17).

The most interesting effect of learning seems to be the changes in the correlation between two thresholds: conditioned and unconditioned (Fig. 18). For the 70dB

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![Graphs](image.png)

**Fig. 14.** Predicted probability of inter-trial responses vs. stage of learning, for 70 dB group (Fig. 14a) and 50 dB group (Fig. 14b).
group this coefficient, being equal to $-0.4$ in the first stage, decreases to almost exactly $-1$ in the second stage and remains unchanged during the next stages. (Precisely, in consecutive stages, it is equal to: $-0.38$, $-0.98$, $-0.99$, $-0.96$, $-0.97$). This effect was not observed for the 50 dB group, for which this coefficient decreases to almost $-0.8$ in the second stage, but increases back to about $-0.5$ in the next stages.

![Diagram](image1.png)

Fig. 15. The difference between the asymptotic value of the excitatory potential and the mean value of the threshold, $H_e$ (Fig. 15a) and $H_u$ (Fig. 15b). For explanations see Fig. 10.

![Diagram](image2.png)

Fig. 16. The measured and theoretical (based on the model) level of performance vs. stage of learning. Filled signs—results for the 70 dB group, empty signs—results for the 50 dB group.
Fig. 17. The sample latency distribution (marked line) and the theoretical latency (unmarked line) distribution. The difference between distributions is caused by including the ITR-probabilities in the theoretical distributions. After vertical shifting by the value $P(\text{ITR})$ the curves are undistinguishable. 
a, group 70 dB, stage 1st, 
b, group 70 dB, stage 5th, 
c, group 50 dB, stage 1st, 
d, group 50 dB, stage 5th. Time is measured in 0.2 s units. For parts c) and d) see next page.

In all stages the correlation coefficient is negative, that is, a negative correlation of thresholds is discovered\(^4\). Moreover, for the 70 dB group, learning leads to linear deterministic relation between thresholds, namely,

$$L_c + L_u = m_{Lc} + m_{Lu} = \text{constant},$$

\(^4\) For the third stage of learning for the 50 dB group, the identification was not successful. In this stage the value of the correlation coefficient reached about +0.5. This result will be verified in future work.
i.e., the greater the value of the conditioned threshold, the smaller the value of the unconditioned one. These two thresholds become linearly, deterministically related and, if confirmed in the next experiments, this effect can be treated as a substantial feature of the avoidance conditioning. The result obtained for the 50 dB group could mean that the avoidance conditioning reflex had not been fully developed for this group.

The interesting presentation of the stimulation and decision process can be done by drawing the excitatory curve and the threshold distribution on the decision plane. The chosen results (for the 1st, 2nd and 5th stages) are presented in Fig. 19 for the 70 dB group and in Fig. 20 for the 50 dB group. On these figures the exci-
Fig. 18. The changes of the correlation coefficient of the conditioned and the unconditioned thresholds. Filled circles — group 70 dB, empty circles — group 50 dB.

Fig. 19. Decision plane, excitatory curve and mean position of decision region for 70 dB group of animals in a, 1st stage, b, 2nd stage, c, 5th stage of the learning. The threshold distributions are marked by their concentration ellipses (for Fig. 19b, c the ellipses should, in fact, be drawn as segments). On the excitatory curve the time parameter values are marked (in 0.2 s units).

tatory curves are drawn as well as the concentration ellipses of the threshold distribution and the “mean positions” of the reaction regions.

It is seen that “in the average” at the first stage of training the unconditioned threshold is reached first and during the stimulation by both stimuli. At the second stage the “conditioned” threshold is, on the average, reached first and just before
the onset of the unconditional stimulation. At the fifth stage the "conditioned" threshold is also reached first, but earlier than in the second stage. The effect of changes of the correlation coefficient of the threshold is visible as changes of the shape of the concentration ellipses.

Figures 21 and 22 similar to Figs. 19 and 20 show the same decision planes with the excitatory curves and the concentration ellipses, but with marked regions corresponding to the different types of the reaction. Only the interiors of the ellipses have been shaded because they include the region which corresponds to about 70% probability. It is visible that the probability of the avoidance reaction increases during learning. Also, the probability of "late avoidance" reaction first increases and then decreases, and the probability of the escape reaction decreases during learning.

CONCLUDING REMARKS

In the paper the linear dynamic, stochastic (LDS) model of the reaction latency in the random threshold (RT) version was applied to analyze the avoidance responding and to the investigation of avoidance learning. Both effects resulted in:
Fig. 21. Division of the excitatory plane into the regions connected to the various types of the reactions for 70 dB groups of animals, in the 1st (Fig. 21a) 2nd (Fig. 21b) and 5th (Fig. 21c) stage of learning. The ellipses denotes ellipses of concentration of the threshold probabilities and corresponds to 70% of all reactions; therefore, only parts of these ellipses have been shaded in adequate regions to show how big is the corresponding probability. Escape-type reaction dominates in the first stage; There is a great contribution of late avoidance reaction in the second and fifth stage of learning in reactions commonly counted as the escape reactions (LA and E). The dominating types of reaction in the fifth stage are avoidances. See also explanation to Fig. 7.

- decreasing the latency (because the second effect of the above is weaker than the first),
- decreasing the probability of inter-trial responses because of the second effect,
- being constant (for the conditioned system) or decreasing (for the unconditioned system) the level of the certain type performance (unconditioned performance and conditioned performance).

The LDS-RT model is a model which explains an evolution of reaction latency. Using the model one can transform the latency distribution into several parameters which have clear biological interpretation. The model assumes that the external stimuli are the inputs to the inertial units whose outputs (excitatory potentials) are compared to the adequate random thresholds. The external reaction is performed when one of the thresholds is exceeded. Several types of reactions are predicted by the model and, beside the "common" escape and avoidance reaction, the "late avoidance" reaction is predicted to exist. The excitatory potentials, the threshold values (visible as the reaction regions) and the regions of various types of reactions can be clearly presented on the decision plane.
The learning process is supposed to consist in changes in the parameters of the reaction model. Avoidance training was analysed by means of the model, and the model parameters were estimated at the consecutive stages into which the learning process was divided. The trade-offs between the two tendencies was observed (which is analogous to that for the escape learning (3)):

- the changes of the parameters of the transformation systems cause decreasing of the latency,
- the changes of the parameters of the decision systems cause increasing of the latency.

Fig. 22. Division of the excitatory plane to the regions connected to the various types of the reactions, for 50 dB groups of animals, in the 1st (Fig. 22a) 2nd (Fig. 22b) and 5th (Fig. 22c) stage of learning. For remarks see explanations of Fig. 21.

It can also be noted that at the first stages of learning, the late-avoidance responses are a great part of the responses commonly treated as being of the escape type.

It can be observed that the correlation of the thresholds (conditioned and unconditioned) is negative. The effect of the avoidance conditioning is in the decrease of the correlation coefficient of the thresholds to minus one; namely, the thresholds become linearly dependent as a result of the conditioning.
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